Section 01

Problem 1

In the year 2001, the United States will host the International Mathematical Olympiad. Let $I, M,$ and O be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What is the largest possible value of the sum $I + M + O$?

$\textbf{(A)}\ 23 \qquad \textbf{(B)}\ 55 \qquad \textbf{(C)}\ 99 \qquad \textbf{(D)}\ 111 \qquad \textbf{(E)}\ 671$

Problem 2

$2000(2000^{2000}) =$

$\textbf{(A)}\ 2000^{2001} \qquad \textbf{(B)}\ 4000^{2000} \qquad \textbf{(C)}\ 2000^{4000} \qquad \textbf{(D)}\ 4,000,000^{2000} \qquad \textbf{(E)}\ 2000^{4,000,000}$

Problem 3

Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

$\textbf{(A)}\ 40 \qquad \textbf{(B)}\ 50 \qquad \textbf{(C)}\ 55 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Problem 4

The Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ starts with two 1's, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 9$

Problem 5

If $|x-2| = p$, where $x < 2$, then $x - p =$

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 2-2p \qquad \textbf{(D)}\ 2p-2 \qquad \textbf{(E)}\ |2p-2|$

Problem 6

Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 119 \qquad \textbf{(D)}\ 180 \qquad \textbf{(E)}\ 231$

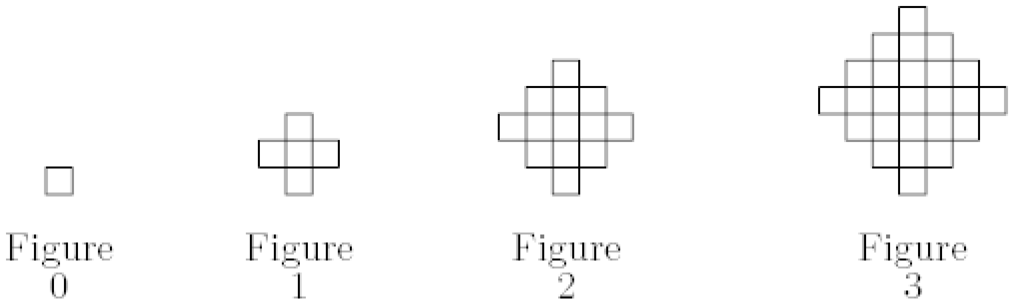
Problem 7

How many positive integers b have the property that $\log_b 729$ is a positive integer?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 8

Figures 0, 1, 2, and 3 consist of 1, 5, 13, and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?



$\textbf{(A)}\ 10401 \qquad \textbf{(B)}\ 19801 \qquad \textbf{(C)}\ 20201 \qquad \textbf{(D)}\ 39801 \qquad \textbf{(E)}\ 40801$

Problem 9

Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were $71, 76, 80, 82,$ and 91. What was the last score Mrs. Walters entered?

$\textbf{(A)}\ 71 \qquad \textbf{(B)}\ 76 \qquad \textbf{(C)}\ 80 \qquad \textbf{(D)}\ 82 \qquad \textbf{(E)}\ 91$

Problem 10

The point $P = (1, 2, 3)$ is reflected in the xy-plane, then its image Q is rotated 180° about the x-axis to produce R, and finally, R is translated 5 units in the positive-y direction to produce S. What are the coordinates of S?

$\textbf{(A)}\ (1,7,-3) \qquad \textbf{(B)}\ (-1,7,-3) \qquad \textbf{(C)}\ (-1,-2,8) \qquad \textbf{(D)}\ (-1,3,3) \qquad \textbf{(E)}\ (1,3,3)$

Problem 11

Two non-zero real numbers, a and b, satisfy $ab = a - b$. Which of the following is a possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ \frac{-1}{2} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ 2$

Problem 12

Let A, M, and C be nonnegative integers such that $A+M+C=12$. What is the maximum value of $A\cdot M\cdot C+A\cdot M+M\cdot C+A\cdot C$?

$\textbf{(A)}\ 62 \qquad \textbf{(B)}\ 72 \qquad \textbf{(C)}\ 92 \qquad \textbf{(D)}\ 102 \qquad \textbf{(E)}\ 112$

Problem 13

One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 14

When the mean, median, and mode of the list

$10, 2, 5, 2, 4, 2, x$

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 17 **(E)** 20

Problem 15

Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

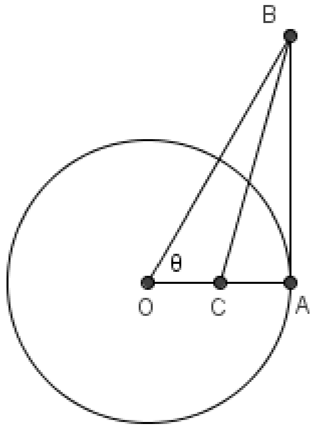
(A) $-1/3$ **(B)** $-1/9$ **(C)** 0 **(D)** 5/9 **(E)** 5/3

Problem 16

A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered $1, 2, \ldots, 17$, the second row $18, 19, \ldots, 34$, and so on down the board. If the board is renumbered so that the left column, top to bottom, is $1, 2, \ldots, 13$, the second column $14, 15, \ldots, 26$ and so on across the board, some squares have the same numbers in both numbering systems. Find the sum of the numbers in these squares (under either system).

(A) 222 **(B)** 333 **(C)** 444 **(D)** 555 **(E)** 666

Problem 17



A circle centered at O has radius 1 and contains the point A. The segment AB is tangent to the circle at A and $\angle AOB = \theta$. If point C lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then $OC =$

$\textbf{(A)}\ \sec^2\theta - \tan\theta \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\cos^2\theta}{1+\sin\theta} \qquad \textbf{(D)}\ \frac{1}{1+\sin\theta} \qquad \textbf{(E)}\ \frac{\sin\theta}{\cos^2\theta}$

Problem 18

In year N, the 300th day of the year is a Tuesday. In year $N+1$, the 200th day is also a Tuesday. On what day of the week did the 100th day of year $N-1$ occur?

$\textbf{(A)}\ \text{Thursday} \qquad \textbf{(B)}\ \text{Friday} \qquad \textbf{(C)}\ \text{Saturday} \qquad \textbf{(D)}\ \text{Sunday} \qquad \textbf{(E)}\ \text{Monday}$

Problem 19

In triangle ABC, $AB = 13$, $BC = 14$, $AC = 15$. Let D denote the midpoint of $\overline{BC}$ and let E denote the intersection of $\overline{BC}$ with the bisector of angle BAC. Which of the following is closest to the area of the triangle ADE?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 2.5 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 3.5 \qquad \textbf{(E)}\ 4$

Problem 20

If x, y, and z are positive numbers satisfying $x + \frac{1}{y} = 4, y + \frac{1}{z} = 1,$ and $z + \frac{1}{x} = \frac{7}{3}$, then what is the value of xyz ?

$\textbf{(A)}\ 2/3 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 4/3 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 7/3$

Problem 21

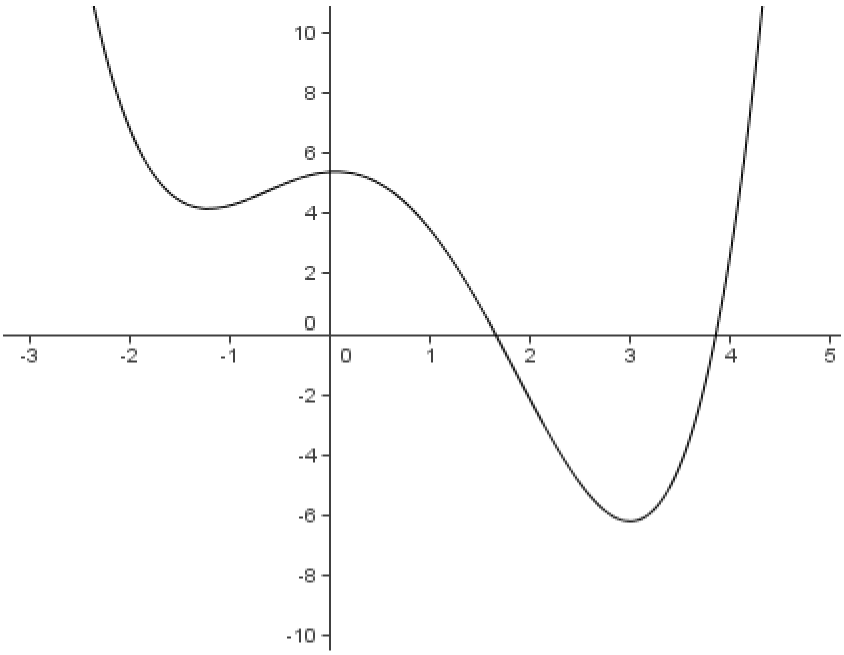
Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

$\textbf{(A)}\ \frac{1}{2m+1} \qquad \textbf{(B)}\ m \qquad \textbf{(C)}\ 1-m \qquad \textbf{(D)}\ \frac{1}{4m} \qquad \textbf{(E)}\ \frac{1}{8m^2}$

Problem 22

The graph below shows a portion of the curve defined by the quartic polynomial $P(x) = x^4 + ax^3 + bx^2 + cx + d$. Which of the following is the smallest?

$\textbf{(A)}\ P(-1)$
$\textbf{(B)}\ \text{The product of the zeros of } P$
$\textbf{(C)}\ \text{The product of the non-real zeros of } P$
$\textbf{(D)}\ \text{The sum of the coefficients of } P$
$\textbf{(E)}\ \text{The sum of the real zeros of } P$

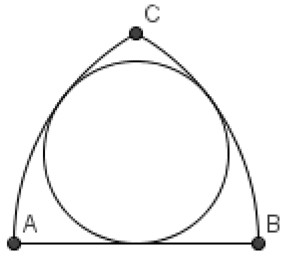


Problem 23

Professor Gamble buys a lottery ticket, which requires that he pick six different integers from 1through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property— the sum of the base-ten logarithms is an integer. What is the probability that Professor Gamble holds the winning ticket?

$\mathrm{(A)}\ 1/5 \qquad \mathrm{(B)}\ 1/4 \qquad \mathrm{(C)}\ 1/3 \qquad \mathrm{(D)}\ 1/2 \qquad \mathrm{(E)}\ 1$

Problem 24



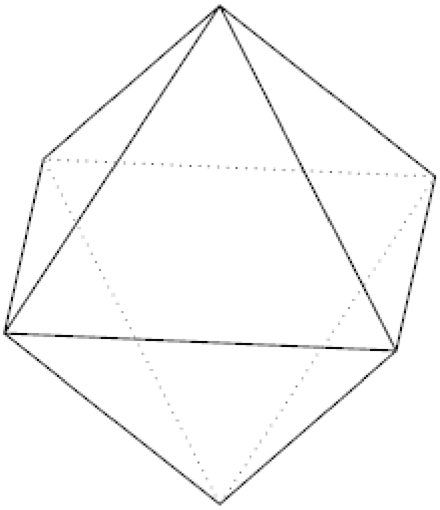
If circular arcs $\overarc{AC}$and $\overarc{BC}$have centers at $B$and A, respectively, then there exists a circle tangent to both $\overarc{AC}$and $\overarc{BC}$, and to $\overline{AB}$. If the length of $\overarc{BC}$is 12, then the circumference of the circle is

$\mathrm{(A)}\ 24 \qquad \mathrm{(B)}\ 25 \qquad \mathrm{(C)}\ 26 \qquad \mathrm{(D)}\ 27 \qquad \mathrm{(E)}\ 28$

Problem 25

Eight congruent equilateral triangles, each of a different color, are used to construct a regular octahedron. How many distinguishable ways are there to construct the octahedron? (Two colored octahedrons are distinguishable if neither can be rotated to look just like the other.)

(A) 210 **(B)** 560 **(C)** 840 **(D)** 1260 **(E)** 1680



Section 02

Problem 1

The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

(A) $2S+3$ (B) $3S+2$ (C) $3S+6$ (D) $2S+6$ (E) $2S+12$

Problem 2

Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23)=6$ and $S(23)=5$. Suppose N is a two-digit number such that $N=P(N)+S(N)$. What is the units digit of N?

(A) 2 (B) 3 (C) 6 (D) 8 (E) 9

Problem 3

The state income tax where Kristin lives is levied at the rate of $p\%$ of the first $\$28000$ of annual income plus $(p+2)\%$ of any amount above $\$28000$. Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

$\text{(A)}\ \$28000 \qquad \text{(B)}\ \$32000 \qquad \text{(C)}\ \$35000 \qquad \text{(D)}\ \$42000 \qquad \text{(E)}\ \$56000$

Problem 4

The mean of three numbers is 10 more than the least of the numbers and 15 less than the greatest. The median of the three numbers is 5. What is their sum?

$\text{(A)}\ 5 \qquad \text{(B)}\ 20 \qquad \text{(C)}\ 25 \qquad \text{(D)}\ 30 \qquad \text{(E)}\ 36$

Problem 5

What is the product of all positive odd integers less than 10000?

$\text{(A)}\ \dfrac{10000!}{(5000!)^2} \qquad \text{(B)}\ \dfrac{10000!}{2^{5000}} \qquad \text{(C)}\ \dfrac{9999!}{2^{5000}} \qquad \text{(D)}\ \dfrac{10000!}{2^{5000} \cdot 5000!} \qquad \text{(E)}\ \dfrac{5000!}{2^{5000}}$

Problem 6

A telephone number has the form ABC-DEF-GHIJ, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

$\text{(A)}\ 4 \qquad \text{(B)}\ 5 \qquad \text{(C)}\ 6 \qquad \text{(D)}\ 7 \qquad \text{(E)}\ 8$

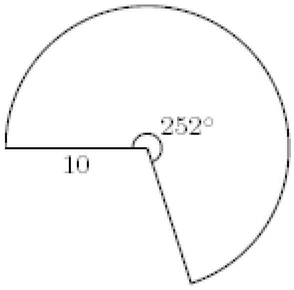
Problem 7

A charity sells 140 benefit tickets for a total of $\$2001$. Some tickets sell for full price (a whole dollar amount), and the rest sells for half price. How much money is raised by the full-price tickets?

(A) $\$782$ (B) $\$986$ (C) $\$1158$ (D) $\$1219$ (E) $\$1449$

Problem 8

Which of the cones listed below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?



(A) A cone with slant height of 10 and radius 6

(B) A cone with height of 10 and radius 6

(C) A cone with slant height of 10 and radius 7

(D) A cone with height of 10 and radius 7

(E) A cone with slant height of 10 and radius 8

Problem 9

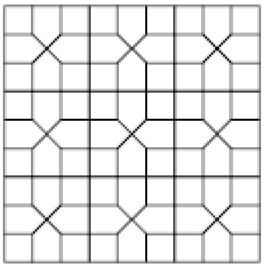
Let f be a function satisfying $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y. If $f(500) = 3$, what is the value of $f(600)$?

(A) 1 (B) 2 (C) $\frac{5}{2}$ (D) 3 (E) $\frac{18}{5}$

Problem 10

The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to

(A) 50 (B) 52 (C) 54 (D) 56 (E) 58



Problem 11

A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

(A) $\frac{3}{10}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{3}{5}$ (E) $\frac{7}{10}$

Problem 12

How many positive integers not exceeding $2001$are multiple of 3or 4but not 5?

(A) 768 (B) 801 (C) 934 (D) 1067 (E) 1167

Problem 13

The parabola with equation $y = ax^2 + bx + c$and vertex $(h, k)$is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

(A) $2b$ (B) $2c$ (C) $2a + 2b$ (D) $2h$ (E) $2k$

Problem 14

Given the nine-sided regular polygon $A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9$, how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $\{A_1, A_2, \ldots, A_9\}$?

(A) 30 (B) 36 (C) 63 (D) 66 (E) 72

Problem 15

An insect lives on the surface of a regular tetrahedron with edges of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge. What is the length of the shortest such trip? (Note: Two edges of a tetrahedron are opposite if they have no common endpoint.)

$\text{(A) } \frac{1}{2}\sqrt{3} \qquad \text{(B) } 1 \qquad \text{(C) } \sqrt{2} \qquad \text{(D) } \frac{3}{2} \qquad \text{(E) } 2$

Problem 16

A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

$\text{(A) } 8! \qquad \text{(B) } 2^8 \cdot 8! \qquad \text{(C) } (8!)^2 \qquad \text{(D) } \frac{16!}{2^8} \qquad \text{(E) } 16!$

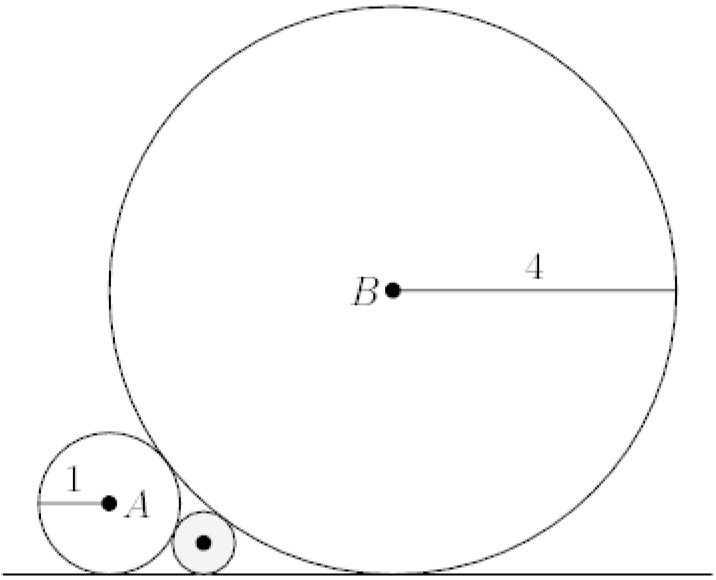
Problem 17

A point P is selected at random from the interior of the pentagon with vertices $A = (0, 2)$, $B = (4, 0)$, $C = (2\pi + 1, 0)$, $D = (2\pi + 1, 4)$, and $E = (0, 4)$. What is the probability that $\angle APB$ is obtuse?

$\text{(A) } \frac{1}{5} \qquad \text{(B) } \frac{1}{4} \qquad \text{(C) } \frac{5}{16} \qquad \text{(D) } \frac{3}{8} \qquad \text{(E) } \frac{1}{2}$

Problem 18

A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. The radius of the third circle is



(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{12}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

Problem 19

The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is 2, what is b?

(A) -11 (B) -10 (C) -9 (D) 1 (E) 5

Problem 20

Points $A = (3, 9)$, $B = (1, 1)$, $C = (5, 3)$, and $D = (a, b)$ lie in the first quadrant and are the vertices of quadrilateral $ABCD$. The quadrilateral formed by joining the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ is a square. What is the sum of the coordinates of point D?

(A) 7 (B) 9 (C) 10 (D) 12 (E) 16

Problem 21

Four positive integers a, b, c, and d have a product of $8!$ and satisfy:

$ab + a + b = 524$
$bc + b + c = 146$
$cd + c + d = 104$

What is $a - d$?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Problem 22

In rectangle $ABCD$, points F and G lie on AB so that $AF = FG = GB$ and E is the midpoint of $\overline{DC}$. Also, $\overline{AC}$ intersects $\overline{EF}$ at H and $\overline{EG}$ at J. The area of the rectangle $ABCD$ is 70. Find the area of triangle EHJ.

(A) $\frac{5}{2}$ (B) $\frac{35}{12}$ (C) 3 (D) $\frac{7}{2}$ (E) $\frac{35}{8}$

Problem 23

A polynomial of degree four with leading coefficient 1 and integer coefficients has two zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

(A) $\frac{1+i\sqrt{11}}{2}$ (B) $\frac{1+i}{2}$ (C) $\frac{1}{2}+i$ (D) $1+\frac{i}{2}$ (E) $\frac{1+i\sqrt{13}}{2}$

Problem 24

In $\triangle ABC$, $\angle ABC = 45^\circ$. Point D is on $\overline{BC}$ so that $2 \cdot BD = CD$ and $\angle DAB = 15^\circ$. Find $\angle ACB$.

(A) 54° (B) 60° (C) 72° (D) 75° (E) 90°

Problem 25

Consider sequences of positive real numbers of the form $x, 2000, y, \cdots$ in which every term after the first is 1 less than the product of its two immediate neighbors. For how many different values of x does the term 2001 appear somewhere in the sequence?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

Section 03

Problem 1

Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$

(A) $\frac{7}{2}$ (B) 4 (C) 5 (D) 7 (E) 13

Problem 2

Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

(A) 15 (B) 34 (C) 43 (D) 51 (E) 138

Problem 3

According to the standard convention for exponentiation, $2^{2^{2^2}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65536.$

If the order in which the exponentiations are performed is changed, how many other values are possible?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

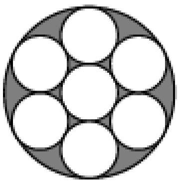
Problem 4

Find the degree measure of an angle whose complement is 25% of its supplement.

(A) 48 (B) 60 (C) 75 (D) 120 (E) 150

Problem 5

Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



$\textbf{(A)}\ \pi \qquad \textbf{(B)}\ 1.5\pi \qquad \textbf{(C)}\ 2\pi \qquad \textbf{(D)}\ 3\pi \qquad \textbf{(E)}\ 3.5\pi$

Problem 6

For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \le m + n$?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}$ infinitely many

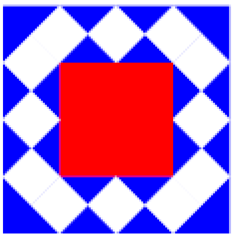
Problem 7

A 45° arc of circle A is equal in length to a 30° arc of circle B. What is the ratio of circle A's area and circle B's area?

$\textbf{(A)}\ 4/9 \qquad \textbf{(B)}\ 2/3 \qquad \textbf{(C)}\ 5/6 \qquad \textbf{(D)}\ 3/2 \qquad \textbf{(E)}\ 9/4$

Problem 8

Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



(A) $B = W$ (B) $W = R$ (C) $B = R$ (D) $3B = 2R$ (E) $2R = W$

Problem 9

Jamal wants to save 30 files onto disks, each with 1.44 MB space. 3 of the files take up 0.8 MB, 12 of the files take up 0.7 MB, and the rest take up 0.4 MB. It is not possible to split a file onto 2 different disks. What is the smallest number of disks needed to store all 30 files?

(A) 12 (B) 13 (C) 14 (D) 15 (E)16

Problem 10

Sarah places four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then pours half the coffee from the first cup to the second and, after stirring thoroughly, pours half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$ (D) $\frac{2}{5}$ (E) $\frac{1}{2}$

Problem 11

Mr. Earl E. Bird gets up every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

(A) 45 (B) 48 (C) 50 (D) 55 (E)58

Problem 12

Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

(A) 0 (B) 1 (C) 2 (D) 4 (E) more than 4

Problem 13

Two different positive numbers a and b each differ from their reciprocals by 1. What is $a + b$?

(A) 1 (B) 2 (C) $\sqrt{5}$ (D) $\sqrt{6}$ (E) 3

Problem 14

For all positive integers n, let $f(n) = \log_{2002} n^2$. Let $N = f(11) + f(13) + f(14)$. Which of the following relations is true?

(A) $N < 1$ (B) $N = 1$ (C) $1 < N < 2$ (D) $N = 2$ (E) $N > 2$

Problem 15

The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

Problem 16

Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

(A) 2/5 (B) 9/20 (C) 1/2 (D) 11/20 (E) 24/25

Problem 17

Several sets of prime numbers, such as $\{7, 83, 421, 659\}$ use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

$\textbf{(A)}\ 193 \qquad \textbf{(B)}\ 207 \qquad \textbf{(C)}\ 225 \qquad \textbf{(D)}\ 252 \qquad \textbf{(E)}\ 447$

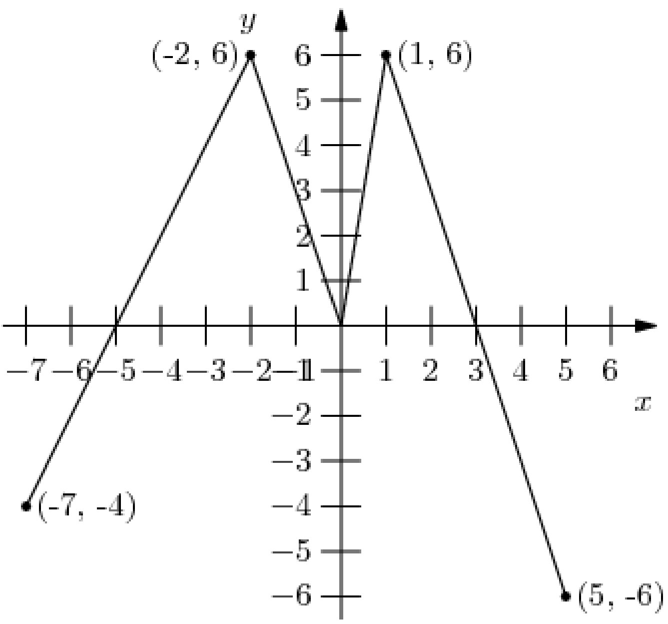
Problem 18

Let C_1 and C_2 be circles defined by $(x-10)^2 + y^2 = 36$ and $(x+15)^2 + y^2 = 81$ respectively. What is the length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q?

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 24$

Problem 19

The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 6$ have?



$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 20

Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

(A) 3 (B) 4 (C) 5 (D) 8 (E) 9

Problem 21

Consider the sequence of numbers: $4, 7, 1, 8, 9, 7, 6, \dots$ For $n > 2$, the n-th term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. The smallest value of n for which $S_n > 10,000$ is:

(A) 1992 (B) 1999 (C) 2001 (D) 2002 (E) 2004

Problem 22

Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60\deg$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?

(A) $\frac{2-\sqrt{2}}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3-\sqrt{3}}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5-\sqrt{5}}{5}$

Problem 23

In triangle ABC, side AC and the perpendicular bisector of BC meet in point D, and BD bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

(A) 14 (B) 21 (C) 28 (D) $14\sqrt{5}$ (E) $28\sqrt{5}$

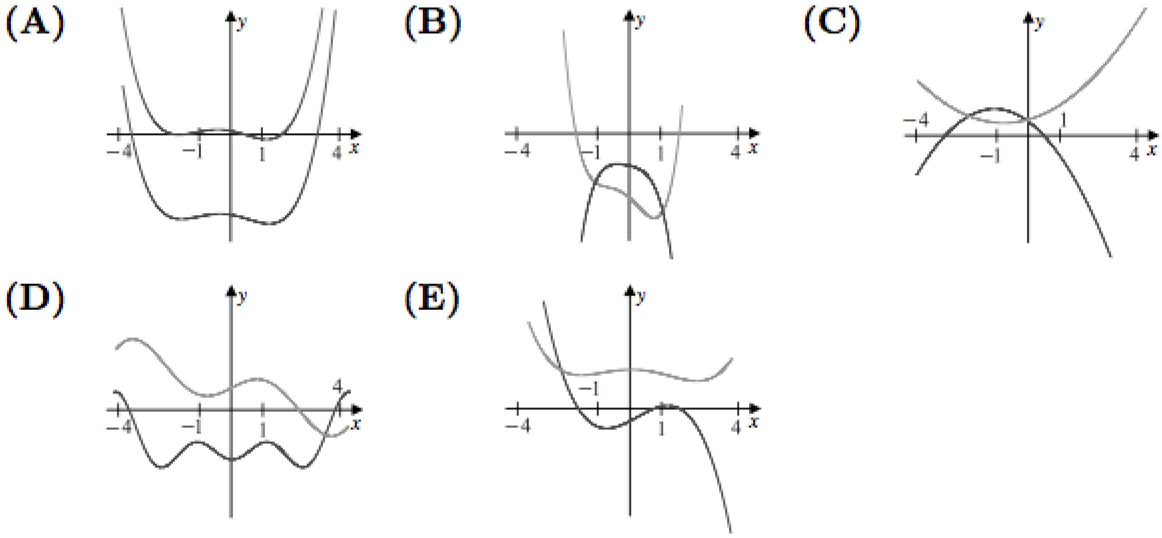
Problem 24

Find the number of ordered pairs of real numbers (a, b) such that $(a + bi)^{2002} = a - bi$.

(A) 1001 (B) 1002 (C) 2001 (D) 2002 (E) 2004

Problem 25

The nonzero coefficients of a polynomial P with real coefficients are all replaced by their mean to form a polynomial Q. Which of the following could be a graph of $y = P(x)$ and $y = Q(x)$ over the interval $-4 \leq x \leq 4$?



Section 04

Problem 1

What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

(A) 0 (B) 1 (C) 2 (D) 2003 (E) 4006

Problem 2

Members of the Rockham Soccer League buy socks and T-shirts. Socks cost \$4 per pair and each T-shirt costs \$5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is \$2366, how many members are in the League?

(A) 77 (B) 91 (C) 143 (D) 182 (E) 286

Problem 3

A solid box is 15cm by 10cm by 8cm. A new solid is formed by removing a cube 3cm on a side from each corner of this box. What percent of the original volume is removed?

$\text{(A)}\ 4.5 \qquad \text{(B)}\ 9 \qquad \text{(C)}\ 12 \qquad \text{(D)}\ 18 \qquad \text{(E)}\ 24$

Problem 4

It takes Mary 30minutes to walk uphill 1km from her home to school, but it takes her only 10 minutes to walk from school to her home along the same route. What is her average speed, in km/hr, for the round trip?

$\text{(A)}\ 3 \qquad \text{(B)}\ 3.125 \qquad \text{(C)}\ 3.5 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 4.5$

Problem 5

The sum of the two 5-digit numbers $AMC10$and $AMC12$is 123422. What is $A+M+C$?

$\text{(A)}\ 10 \qquad \text{(B)}\ 11 \qquad \text{(C)}\ 12 \qquad \text{(D)}\ 13 \qquad \text{(E)}\ 14$

Problem 6

Define $x\heartsuit y$to be $|x-y|$for all real numbers $x$and y. Which of the following statements is not true?

$\text{(A)}\ x\heartsuit y = y\heartsuit x$for all $x$and y

$\text{(B)}\ 2(x\heartsuit y) = (2x)\heartsuit (2y)$for all $x$and y

$\text{(C)}\ x\heartsuit 0 = x$for all x

$\text{(D)}\ x\heartsuit x = 0$for all x

$\text{(E)}\ x\heartsuit y > 0$if $x\neq y$

Problem 7

How many non-congruent triangles with perimeter 7have integer side lengths?

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 5$

Problem 8

What is the probability that a randomly drawn positive factor of 60 is less than 7?

$\textbf{(A)}\ \frac{1}{10}\qquad\textbf{(B)}\ \frac{1}{6}\qquad\textbf{(C)}\ \frac{1}{4}\qquad\textbf{(D)}\ \frac{1}{3}\qquad\textbf{(E)}\ \frac{1}{2}$

Problem 9

A set S of points in the xy-plane is symmetric about the orgin, both coordinate axes, and the line $y=x$. If $(2,3)$ is in S, what is the smallest number of points in S?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 4\qquad\textbf{(D)}\ 8\qquad\textbf{(E)}\ 16$

Problem 10

Al, Bert, and Carl are the winners of a school drawing for a pile of Halloween candy, which they are to divide in a ratio of $3:2:1$, respectively. Due to some confusion they come at different times to claim their prizes, and each assumes he is the first to arrive. If each takes what he believes to be the correct share of candy, what fraction of the candy goes unclaimed?

$\textbf{(A)}\ \frac{1}{18}\qquad\textbf{(B)}\ \frac{1}{6}\qquad\textbf{(C)}\ \frac{2}{9}\qquad\textbf{(D)}\ \frac{5}{18}\qquad\textbf{(E)}\ \frac{5}{12}$

Problem 11

A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B the area of the circle circumscribed around the triangle. Find A/B.

$\textbf{(A)}\ \frac{9}{16}\qquad\textbf{(B)}\ \frac{3}{4}\qquad\textbf{(C)}\ \frac{27}{32}\qquad\textbf{(D)}\ \frac{3\sqrt{6}}{8}\qquad\textbf{(E)}\ 1$

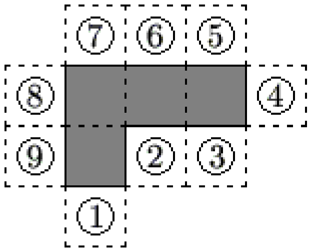
Problem 12

Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

$\textbf{(A)}\ 8\qquad\textbf{(B)}\ 9\qquad\textbf{(C)}\ 10\qquad\textbf{(D)}\ 11\qquad\textbf{(E)}\ 12$

Problem 13

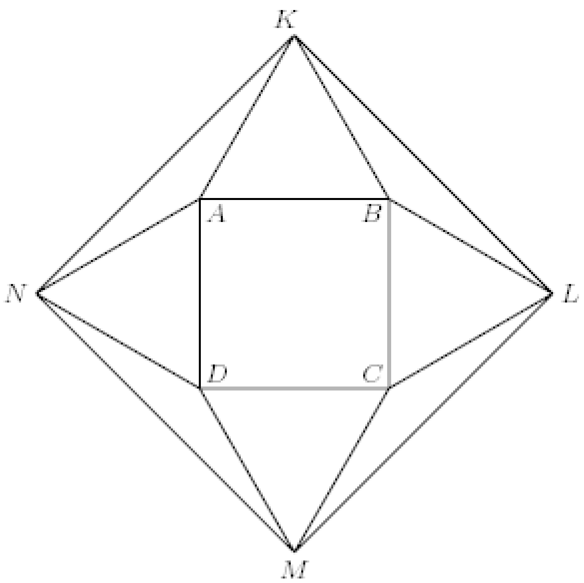
The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 14

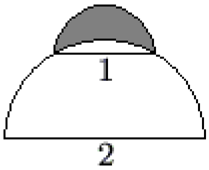
Points $K, L, M,$ and N lie in the plane of the square $ABCD$ such that AKB, BLC, CMD, and DNA are equilateral triangles. If $ABCD$ has an area of 16, find the area of $KLMN$.



(A) 32 (B) $16+16\sqrt{3}$ (C) 48 (D) $32+16\sqrt{3}$ (E) 64

Problem 15

A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.



(A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ (C) $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ (D) $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$ (E) $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

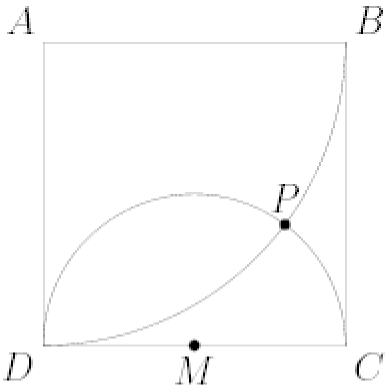
Problem 16

A point P is chosen at random in the interior of equilateral triangle ABC. What is the probability that $\triangle ABP$ has a greater area than each of $\triangle ACP$ and $\triangle BCP$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

Problem 17

Square $ABCD$ has sides of length 4, and M is the midpoint of $\overline{CD}$. A circle with radius 2 and center M intersects a circle with radius 4 and center A at points P and D. What is the distance from P to $\overline{AD}$?



(A) 3 **(B)** $\frac{16}{5}$ **(C)** $\frac{13}{4}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{7}{2}$